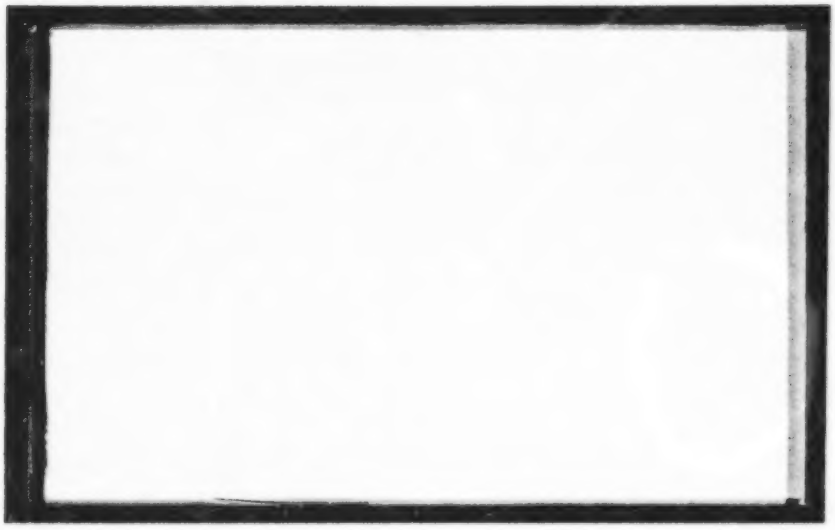
UNIVERSITÉ DE MONTRÉAL



CENTRE DE RECHERCHE SUR LES TRANSPORTS

CASE POSTALE 6128, SUCCURSALE A, MONTRÉAL, QUÉBEC, H3C 3J7

IMPACT OF PASSENGER TRAIN OPERATION ON FREIGHT TRAIN OPERATION

by

G. Raghuram*

* Scholar-in-Residence, Queen's University
Asst-Professor of Management Science, Tulane University

Presented at the Management Science Workshop on Transportation, Queen's University, 84-06-22 and at the Centre de recherche sur les transports of the Université de Montréal, 84-07-18.

Also published as working paper No. 84-16 of the School of Business, Queen's University, Kingston, Ontario.

Université de Montréal
Centre de recherche sur les transports - Publication #370
Septembre 1984

Dépôt légal - 1er trimestre 1985

ABSTRACT

We consider a railroad which has the potential to be saturated to capacity by just running freight trains. Profit considerations require that only freight trains be run. However, other considerations (service, image, politics, etc.) require the running of passenger trains, usually at speeds and priority higher than those of freight trains. Hence, a proper mix of freight trains and passenger trains needs to be determined. To determine such a mix, we need to have an idea of the equivalence between passenger trains and freight trains, and the factors that influence such an equivalence. This paper presents an analytical model which develops such an equivalence and helps understand the factors behind the equivalence.

ACKNOWLEDGEMENTS

I am grateful to the Scholars-in-Residence program of the School of Business, Queen's University and the School of Business, Tulane University, for providing the necessary support for this research.

I am grateful to the Indian Institute of Management, Ahmedabad, where the early ideas of this research were developed.

I am also grateful to the many Indian Railway Officers, discussions with whom helped me understand the problem.

1. The Problem

We consider a railroad which has the potential to be saturated to capacity by just running freight trains. Profit considerations require that only freight trains be run. However, other considerations (service, image, politics, etc.) require the running of passenger trains, usually at speeds and priority higher than those of the freight trains. Hence, a proper mix of freight trains and passenger trains needs to be determined. To determine such a mix, we need to have an idea of the equivalence between passenger trains and freight trains, and the factors that influence such an equivalence. This chapter presents an analytical model which develops such an equivalence and helps understand the factors behind the equivalence.

Similar research in analytical modeling is limited. Railroads generally use detailed simulation models - not to directly make an equivalence estimation, but more generally to measure capacity as a macro-indicator of the number of trains that can be run in a section. Ove Frank developed an analytical model for evaluating traffic capacity on a single track railway with only meets between trains [Frank, 1966]. Petersen developed an analytical model of the mean running time for trains on a single-track railway, as part of a larger railway planning model [Petersen, 1977]. Singh came closest to making an analytic evaluation of the equivalent loss [Singh, 1965], but had some errors in the model as identified by Raghuram [Raghuram, 1979]. Petersen and Taylor have done more work on dispatch policies for optimal overtakes/meets and to avoid line-blockage [Petersen and Taylor, 1982, 1983]. There is some related literature on time-tabling and the debate on whether more fast

passenger trains should be run at the cost of slower, but more profitable freight trains. Some of them came out in connection with a 'high-speed' symposium organized by the International Railway Congress Association [Maternini, 1969], [Pottholf, 1968], [Stahli, 1969], and [Vorobjev, 1968].

The analytical model is described in Section 2, explaining the approach to determine the freight train capacity loss due to the running of a passenger train. Sections 3, 4 and 5 evaluate a lower bound and the actual loss suffered by the freight train capacity. In Section 6, we perform comparative statics on the loss function. Section 7 examines the situation when more than one passenger train is to be introduced. Section 8 provides the conclusions and directions for future research.

2. The Model

We first define a 'section' of railroad track as being between two points such that trains do not originate or terminate anywhere between the two points. We consider a double track section of a railroad with each track assigned to carry traffic in one direction. This scenario does not significantly reduce the applicability of our model since most high density routes are double track and consequently we address ourselves to a major share of the freight traffic. With this scenario the problem is decomposable, making the analysis identical for each direction. Hence, we need look at only a single track section with all trains moving in the same direction.

We start with the scenario where the section is saturated to capacity with freight trains. Then we pose the question -- if we decide to run one passenger train, how many freight trains do we lose? (The case of running more than one passenger train is discussed in Section 7.) The loss is a result of delays that the freight trains incur while

being overtaken by the passenger train, which has priority over the freight trains.

The overtakes are arranged at sidings which occur at regular intervals in the section. Sidings have as many tracks as would be necessary to arrange overtakes. The length of track between sidings is further divided into blocks, each block permitting one train in its limits. We assume that blocks are of equal length and that there are equal numbers of blocks between any two consecutive sidings. Each block has a length d, and the number of blocks between consecutive sidings is n.

We assume that all freight trains run at a speed f and all passenger trains run at a speed p, $p > f$. We define

$$r = \frac{f}{p} \qquad (1)$$

that is, r is the ratio of the speed of the slow freight trains to that of the faster passenger trains. We further assume that stopping times for trains in a section, other than overtake related, are small enough to be neglected. We also neglect any additional time taken for acceleration or deceleration.

We define a Schedule S as one which specifies the departure times of all the trains from each of the sidings. It follows that a schedule automatically specifies the sidings at which overtakes occur. We state two principles followed hierarchically in selecting a schedule :

<u>First Principle</u> : A passenger train should never have to stop to let a freight train move into a siding.

<u>Second Principle</u> : The number of freight trains run should be maximized, subject to running the required number of passenger trains.

Trains are depicted as linear paths in a graph with a time-distance coordinate axis (Figure 1). Freight train paths would be linear between two sidings and could be linear on an entire section. They would be piece-wise linear if the trains have to stop at a siding to be overtaken. Passenger train paths have to be strictly linear, because of the priority during overtakes based on the first principle. Figure 1 shows a time-distance graph with only freight trains F_0, F_1,...,etc. over a section with sidings S_0, S_1,...,etc.

When only freight trains are run, following the second principle, the headway between freight trains should be the minimum which is $\frac{d}{f}$. If the minimum headway between any two consecutive freight trains is disrupted due to the running of a passenger train, then an equivalent loss of freight trains occurs. For example, if the actual headway is H, then the equivalent loss

$$L = \frac{H}{d/f} - 1 = \frac{fH}{d} - 1 \qquad (2)$$

3. A Lower Bound for the Loss

In order to determine a lower bound for the loss, we need a lower bound for the headway caused by the introduction of a passenger train.

Let F_0 be the freight train immediately following the passenger train P_1 at the starting siding S_0 (Figure 2). F_1, F_2,..., etc. are freight trains that are ahead of P_1 at S_0, and which will subsequently be overtaken by P_1.

At S_0 the headway between F_2 and F_1 is $\frac{d}{f}$. But the headway between F_1 and P_1 is governed by the first principle which requires that P_1 just reach the block before the next siding S_1, when F_1 has

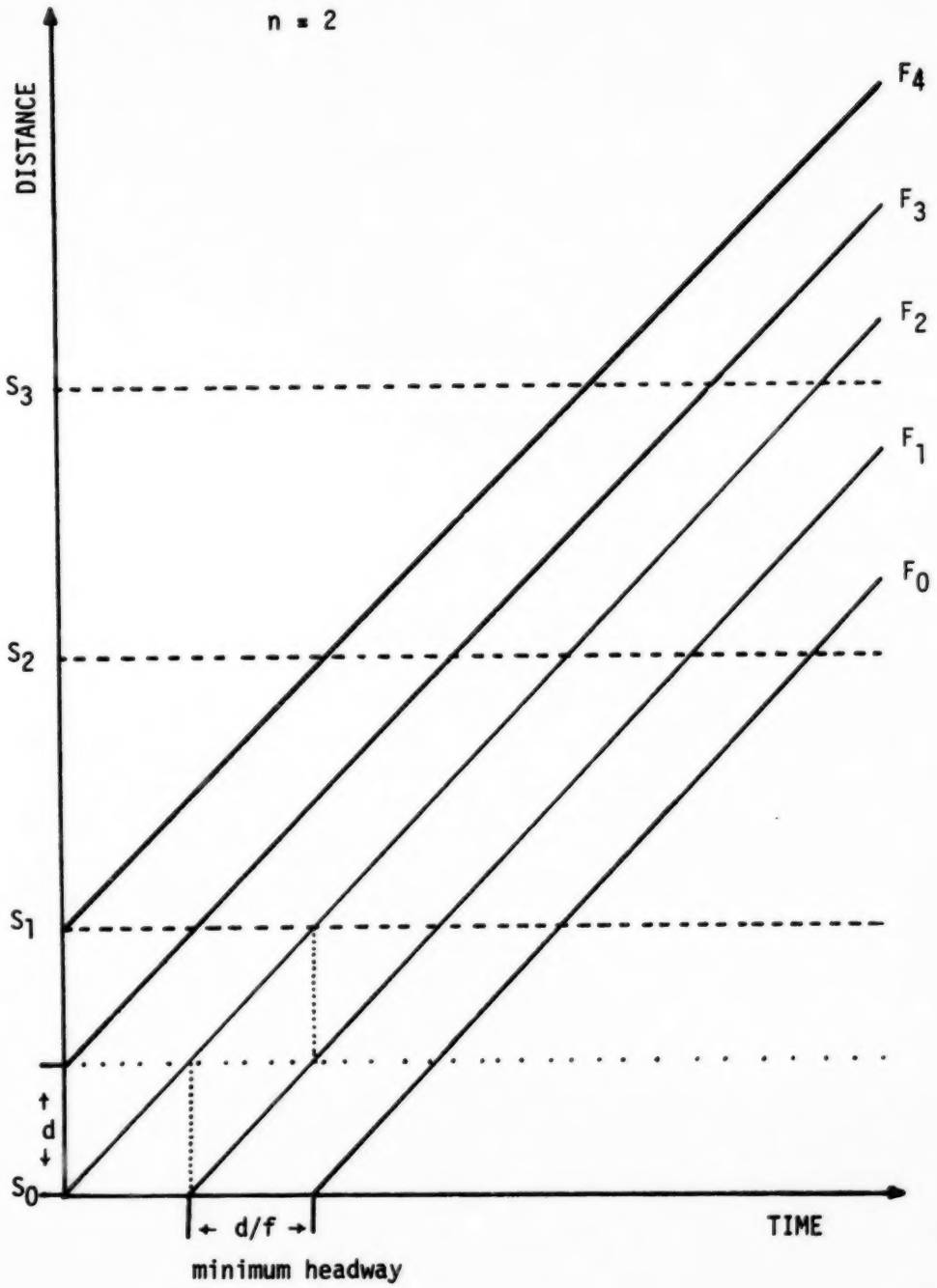


Figure 1 : Trains as a Linear Graph

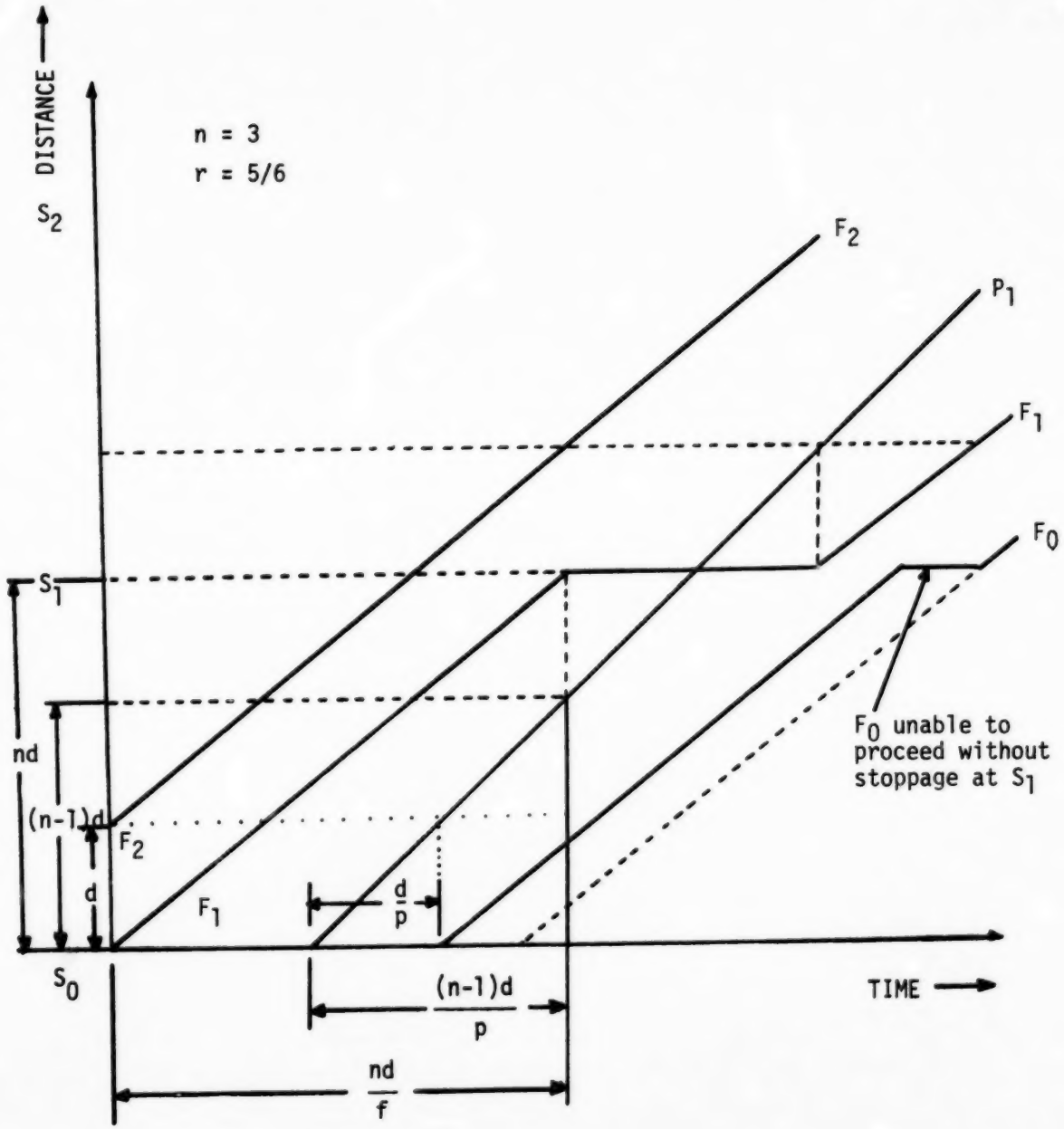


Figure 2 : Overtake of a Freight Train by a Passenger Train

pulled into S_1. So the minimum headway

$$\text{between } F_1 \text{ and } P_1 \text{ at } S_0 = \frac{nd}{f} - \frac{(n-1)d}{p} \qquad (3)$$

where $\frac{nd}{f}$ is the time taken by F_1 to go from S_0 to S_1 and $\frac{(n-1)d}{p}$ is the time taken by P_1 to go from S_0 to a block short of S_1. Without loss of generality, the right hand side of (5.3) can be interpreted as the minimum headway between a freight train and a following passenger train at any siding such that the freight train can make it to the next siding without violating the first principle.

The headway between P_1 and F_0 can be a minimum of $\frac{d}{p}$, which will enable F_0 to travel to S_1 without any problem. But then, can F_0 continue its journey behind F_1 without any further stoppage at S_1, where F_1 is overtaken by P_1? If there does have to be a stoppage, F_0 might as well be scheduled later from S_0 (as seen in Figure 2). Similarly, can F_1 proceed with minimum headway after P_1 from S_1 or will its departure be affected by future overtakes of freight trains by P_1. If the latter, then the departure of F_0 could again be affected. It seems entirely possible then, that future overtakes by P_1 would affect the schedule of F_0 and effectively delay its departure from S_0. The question then is - how many overtakes and how many sidings do we need to consider before we can fix the departure of F_0 from S_0. Once we obtain this and can fix the departure of F_0 from S_0, we can calculate the headway which determines the actual loss.

From the above discussion we note that the paths of already overtaken freight trains can be considered as being linear (i.e. they have a non-stop run after being overtaken), without affecting any of our analysis. Such a consideration proves useful while proving proposition one in Section 4.

Assuming that all will go well for F_0 at later sidings, we can determine the lower bound for the headway H_{LB}, between F_1 and F_0 at S_0, and consequently the lower bound for the loss L_{LB}.

$$H_{LB} = (\text{Minimum Headway between } F_1 \text{ and } P_1) + (\text{Minimum Headway between } P_1 \text{ and } F_0)$$

$$= \frac{nd}{f} - \frac{(n-1)d}{p} + \frac{d}{p}$$

$$= \frac{nd}{f} - \frac{(n-2)d}{p} . \qquad (4)$$

So, the lower bound for the loss in freight trains due to the introduction of P_1

$$L_{LB} = \frac{H_{LB}}{\frac{d}{f}} - 1$$

$$= n - \frac{(n-2)f}{p} - 1$$

$$= (n-1) - (n-2)r. \qquad (5)$$

4. An Upper Bound for the Loss

In order to determine an upper bound for the loss, we need to examine headways caused at different sidings where overtakes occur. We define H(t,S) to be the headway between a train F_t (just about to be overtaken by P_1) and F_{t-1} (just overtaken by P_1) based on a schedule S. We define BOT(t,s) and AOT(t,S) called Before Overtake Time and After Overtake Time, to represent the times spent by a freight train t before and after its overtake, respectively as per schedule S (Figure 3). Since the minimum headway between two consecutive freight trains

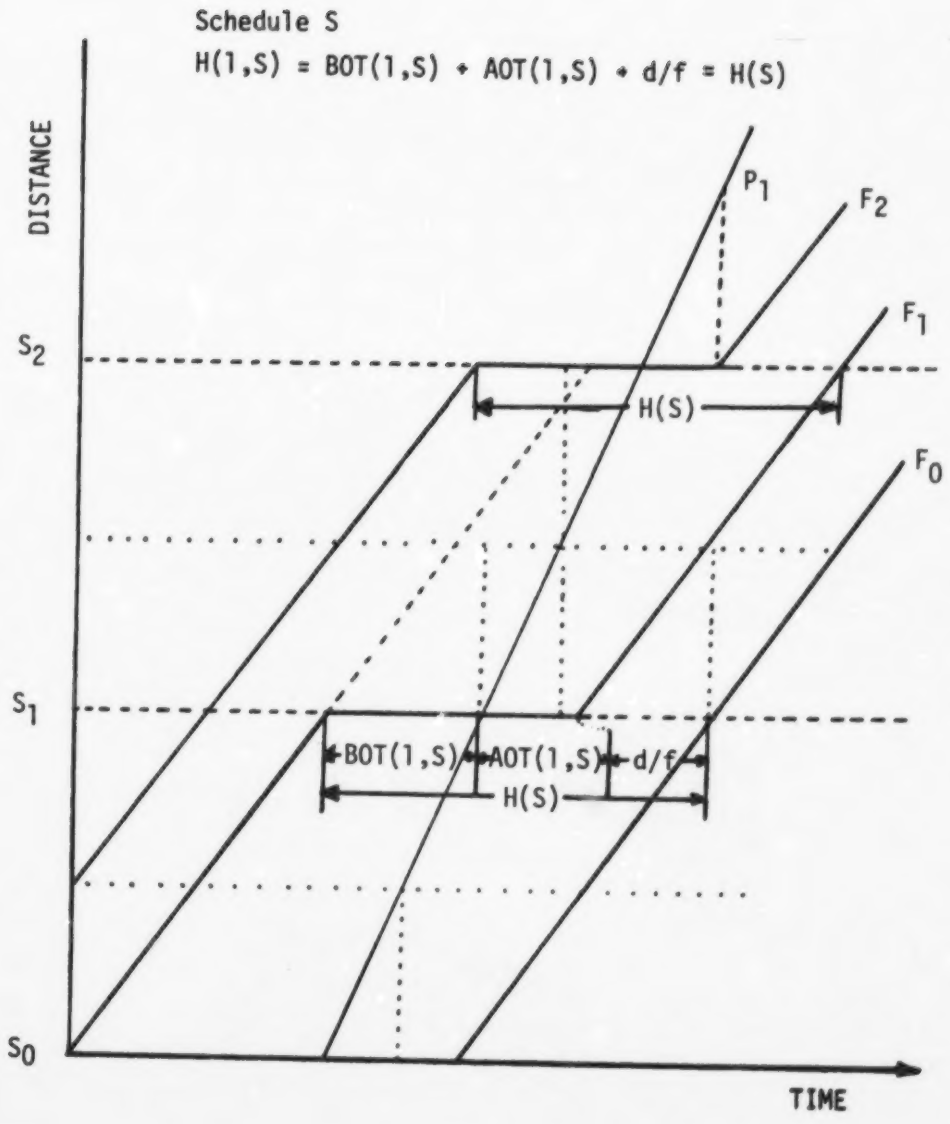


Figure 3 : BOT, AOT, and Headway H

without interference by a passenger train is $\frac{d}{f}$, we have

$$H(t,S) = BOT(t,S) + AOT(T,S) + \frac{d}{f} \qquad (6)$$

We now make the following propositions for a schedule S' satisfying the two principles stated in Section 2.

Proposition 1 : For a given schedule S'.

$H(1,S') = H(2,S') = \ldots H(t,S') = H(S')$

Proof : Every overtake represents a freight train path moving from a to-be-overtaken set of freight trains to an already-overtaken set of freight trains. All freight train paths are parallel and within each set, the train paths are linear with a headway of $\frac{d}{f}$. Consequently, any shift of a path from one set to the other will maintain a constant headway between the two sets, for a given schedule. Q.E.D.

Proposition 2 : For a given schedule S',

$BOT(t,S') + AOT(t,S')$ is a constant independent of t.

Proof : Follows from (6) and proposition one, since then we have

$$BOT(t,S') + AOT(t,S') = H(S') - \frac{d}{f}$$ Q.E.D.

Proposition 3 : S' satisfies the second principle (i.e. S' maximizes the number of freight trains subject to running the required number of passenger trains)

$\Leftrightarrow$ for every schedule S, $H(S') < H(S)$

Proof : Maximizing the number of freight trains is equivalent to minimizing the loss of freight trains. Hence, $L(S') < L(S)$ for every schedule S.

But from (2) we have

$L(S) = \frac{fH(S)}{d} - 1$, where f, d are independent of the schedule and positive. It follows $H(S') < H(S)$ for every schedule S. Q.E.D.

Proposition 4 : For a given schedule S',

$AOT(t,S') = \frac{d}{p}$ for at least one t.

Proof : Clearly $AOT(t,S')$ cannot be less than $\frac{d}{f}$ which is the minimum headway for a train to follow a passenger train. Now, suppose the proposition is not true, then

$AOT(t,S') - \frac{d}{f} => 0$ for every t

$<=> \min_t D\ (t,S') = D\ (S') > 0$

where $D\ (t,S') = AOT\ (t,S') - \frac{d}{f}$

This means that all 'already-overtaken' trains can be advanced in schedule by D(S'), (without affecting BOT(t,S')), thereby obtaining a new schedule S" such that $H(S") < H(S')$. By proposition three, this contradicts the fact that S' satisfies the second principle. Q.E.D.

Proposition 5 : For a given schedule S',

$AOT(t,S') = \frac{d}{p} <=> BOT(t,S')$ is maximum.

Proof : The proposition follows directly from the fact that $\min AOT(t,S') = \frac{d}{p}$ and from proposition two Q.E.D.

Proposition 6 : Minimizing $H(S)$ is equivalent to

$\min_S$ Minimizing $\{\max_t BOT(t,s)\}$

Proof : From (6) and propositions four and five, we can write

$$H(S) = \max_t BOT(t,s) + \frac{d}{p} + \frac{d}{f}$$

Since $\frac{d}{p}$ and $\frac{d}{f}$ are independent of S, proposition six follows. Q.E.D.

Proposition 7 : $\min_S \{\max_t BOT(t,S)\} < \frac{nd}{f} - (n-1)\frac{d}{p}$

Suppose the proposition is not true, then there exists train t^1 such that

$$\min BOT(t',S) = \frac{nd}{f} - (n-1)\frac{d}{p}$$

From (3), we know that the train t' can move to a siding ahead to reduce the before overtake time, thereby contradicting the minimization over S. Q.E.D.

So proposition seven gives the upper bound for $BOT(BOT_{UB})$ under a schedule satisfying both the principles of Section 5.2. Using propositions five and two we have the upper bound for BOT and AOT as

$$(BOT + AOT)_{UB} = BOT_{UB} + \frac{d}{p}$$

$$= \frac{nd}{f} - (n-1)\frac{d}{p} + \frac{d}{p}$$

So, from (6) the upper bound for the headway

$$H_{UB} = \frac{nd}{f} - (n-1)\frac{d}{p} + \frac{d}{p} + \frac{d}{f}$$

$$= (n+1)\frac{d}{f} - (n-2)\frac{d}{p} \qquad (7)$$

So, the upper bound for the loss in freight trains due to the introduction of a passenger train P_1 is

$$L_{UB} = \frac{H_{UB}}{\frac{d}{f}} - 1$$

$$= (n+1) - (n-2)\frac{f}{p} - 1$$

$$= n - (n-2)r \qquad (8)$$

5. The Actual Loss

In Section 4, we determined the upper bound for the loss by obtaining an upper bound for the BOT - a BOT that actually does not occur (proposition seven). If instead, we determine the maximum BOT that occurs, then we can obtain the actual headway (using propositions five and two) and consequently the actual loss.

We first observe that if for a schedule S,

$$BOT(t,S) = BOT(t + m, s) \qquad (9)$$

then the overtake pattern is repetitive every m overtakes. If m is the minimum number of overtakes for which the pattern is repetitive, then

$$BOT(t,S) = BOT(t + i, S) \text{ for every } t,\ 0 < i < m \qquad (10)$$

Hence, to determine the maximum BOT that occurs, we need to look at m consecutive overtakes only.

We first obtain the value of m. Let us say that the repetitive pattern of m overtakes occurs over N sidings. This means that over the nN blocks, a freight train takes $m\frac{d}{f}$ more time than a passenger train (Figure 4).

$$\text{Hence, } \frac{Nnd}{f} = \frac{Nnd}{p} + \frac{md}{f}$$

$$\frac{m}{N} = n\left(1 - \frac{f}{p}\right)$$

$$= n(1-r)$$

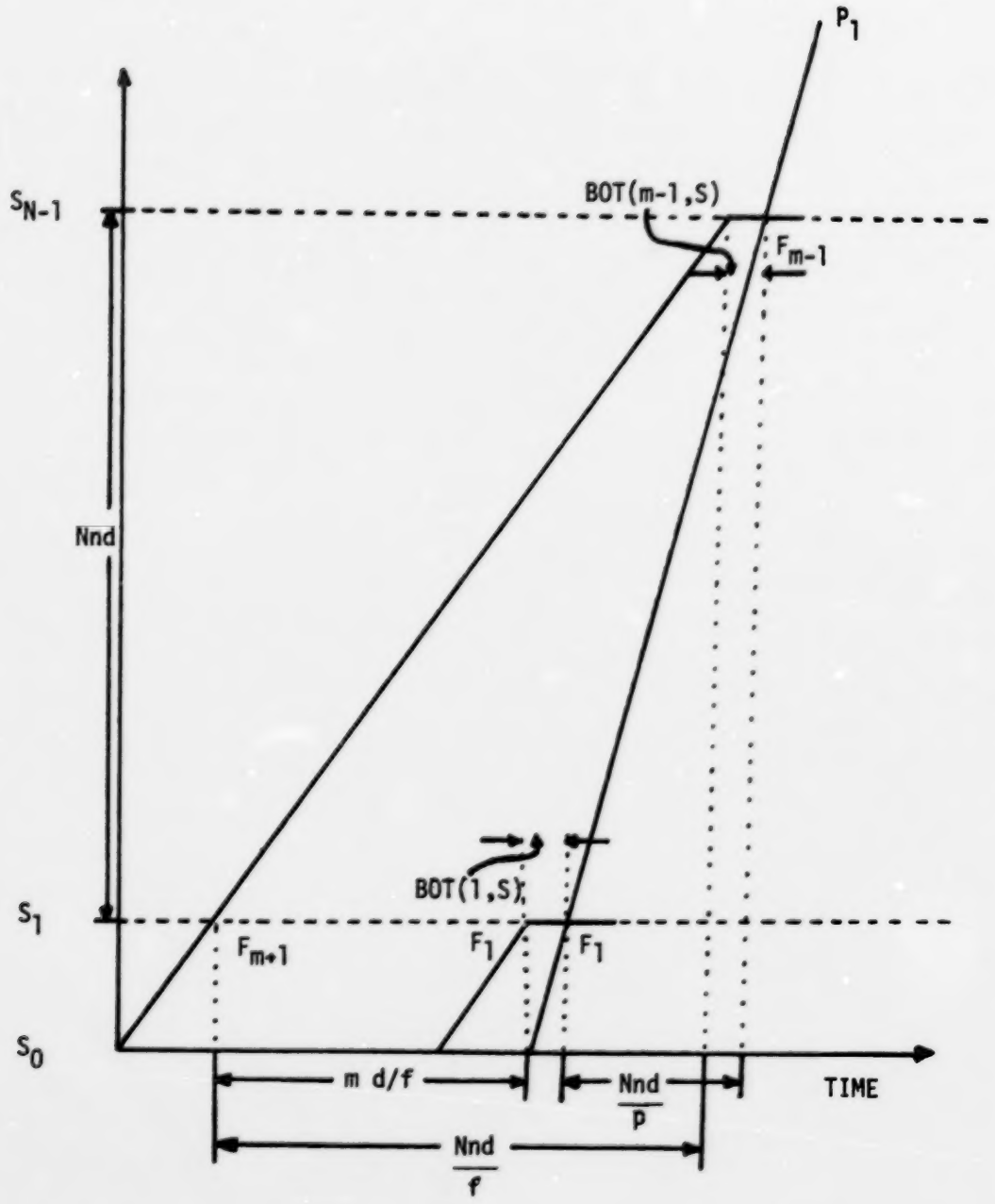


Figure 4 : m Overtakes in N sidings

Since m,N should be integral, we evaluate n(1-r) and obtain a numerical expression whose numerator equals m and denominator equals N. But then, the values of m and N would not be unique. However, since we need the smallest repetitive pattern, we can include the constraint that m,N must be relatively prime

$$\text{So, } \frac{m}{N} = n(1-r) \text{ m,N relatively prime.} \qquad (11)$$

For example, if n = 3 and r = 5/6, then n(1-r) = 3(1-5/6) = 3/6 = 1/2. We consider m = 1 and N = 2 and not m = 3 and N = 6 since we are looking for the smallest repetitive pattern.

We examine the repetitive pattern from Siding S_1 to Siding S_{N+1} to obtain the maximum BOT that occurs (Figure 4). At S_1, F_1 is overtaken by P_1 and at S_{n+1}, F_{m+1} is overtaken.

If we look at the path of F_{m+1} : at S_1, it arrives $\frac{d}{p} + \frac{md}{f}$ time units before P_1. At S_{N+1}, it arrives $\frac{d}{p}$ time units before P_1.

So, at any siding S_q, it is $\frac{d}{p} + \frac{(N+1-q)}{N}\left(\frac{md}{f}\right)$ time units before P_1.

Trains after F_{m+1}, but before P_1 will be correspondingly $\frac{d}{f}$, $\frac{2d}{f}$, ...,etc. time units after F_{m+1}. Let us say in general $\frac{zd}{f}$ units after F_{m+1} where Z = # of trains after F_{m+1}. We note that this is valid only for trains before P_1. So they will be in general

$$\frac{d}{p} + \frac{(N+1-q)}{N}\frac{md}{f} - \frac{zd}{f} \text{ time units before } P_1$$

$$= \frac{d}{p} + \frac{(N+1-q)m - zN}{N}\left(\frac{d}{f}\right) \text{ time units before } P_1$$

$$= \frac{d}{p} + \frac{X}{N}\left(\frac{d}{f}\right) \text{ time units before } P_1 \qquad (12)$$

We note that in general, (12) need not be a BOT. We make now the following observations.

1. $X > 0$

   This follows directly from the fact that the above expression is valid only for trains before P_1.

2. X is an integer.

   Follows from the fact that $X = (N+1-q)m - Zn$ where N (# of sidings in the repetitive pattern), q (siding number), m (# of trains overtaken in the repetitive pattern), Z (# of trains after F_{m+1}) are all integers.

3. If (12) is to be a BOT, then $X < m$.

   Since overtakes are scheduled only if the headway between a freight train and a passenger train is less than (3). We have the following condition for (12) to be a BOT.

$$\frac{d}{p} + \frac{X}{N}\left(\frac{d}{f}\right) < \frac{nd}{f} - \frac{(n-1)d}{p}$$

$$<=> \quad \frac{X}{N}\left(\frac{d}{f}\right) < \frac{nd}{f} - \frac{nd}{p}$$

$$<=> \quad \frac{X}{N} < n(1-r) \qquad \text{(from 11)}$$

$$<=> \quad \frac{X}{N} < \frac{m}{N}$$

$$<=> \quad X < m$$

Based on the observations above, we have the general expression for a BOT as follows :

$$BOT(t,S) = \frac{d}{p} + \frac{X}{N}\left(\frac{d}{f}\right)$$

$$0 <= X < m, \ X \text{ integer} \qquad (13)$$

Proposition 8 : $\max_t BOT(t,S) = \frac{d}{p} + \frac{m-1}{N}(\frac{d}{f})$ (14)

Proof : From (13), we know that there are m feasible values for BOT(t,S), the maximum being

$$\frac{d}{p} + \frac{(m-1)}{N}\left(\frac{d}{f}\right)$$

From (10), we know that there are m distinct values for BOT(t,S).

Consequently, the proposition is true. Q.E.D.

From proposition eight, we have the actual headway H_A between a pair of consecutive freight trains caused by the introduction of one passenger train as

$$\begin{aligned} H_A &= MAX_t BOT(T,S) + \frac{d}{p} + \frac{d}{f} \\ &= \frac{d}{p} + \frac{m-1}{N}\left(\frac{d}{f}\right) + \frac{d}{p} + \frac{d}{f} \\ &= \frac{zd}{f} + \frac{(m+N-1)}{N}\left(\frac{d}{f}\right) \end{aligned}$$

So, actual loss in freight trains due to the introduction of a passenger train is

$$\begin{aligned} L_A &= \frac{H_A}{\frac{d}{f}} - 1 \\ &= 2r + \frac{m-1}{N} \qquad (15) \\ &= 2r + n(1-r) - \frac{1}{N} \\ &= (n - \frac{1}{N}) - (n-2)r \qquad (17) \end{aligned}$$

6. Comparative Statics

We can obtain a general idea of how the low function varies as a function of the number of blocks between two sidings (n), or as a function of the ratio of the speed of the freight train to the speed of the passenger train (r). The actual loss (L_A) is a function of m and N which are not well-defined functions of n and r as in 11). Hence, comparative statics on L_A cannot be performed. But then, L_{UB} and L_{LB} are well-defined function of n and r, and we can get comparative statics results for them. Though these results need not be valid for L_A, they at least indicate a general idea.

$$\frac{\partial L_{UB}}{\partial n} = \frac{\partial L_{LB}}{\partial n} = (1-r) \tag{18}$$

$$\frac{\partial L_{UB}}{\partial r} = \frac{\partial L_{LB}}{\partial r} = (2-n) \tag{19}$$

Loss as a Function of n

Since 1-r is always positive, the actual loss increases as n inreases, in general. There are exceptions to this, for example : let

$$r = \frac{2}{3} \text{ and n vary 1 to 7.}$$

$$n = 1,2,3,4,5,6,7,\ldots$$

$$L_A = \frac{4}{3}, \frac{5}{3}, \frac{4}{3}, \frac{7}{3}, \frac{8}{3}, \frac{7}{3}, \frac{10}{3},\ldots$$

Even though L_A increases in general, (from $\frac{4}{3}$ to $\frac{10}{3}$) there are two dips, when n goes 2 to 3 and when it goes from 5 to 6. Figures 5 and 6 demonstrate the case for n = 2 and n = 3.

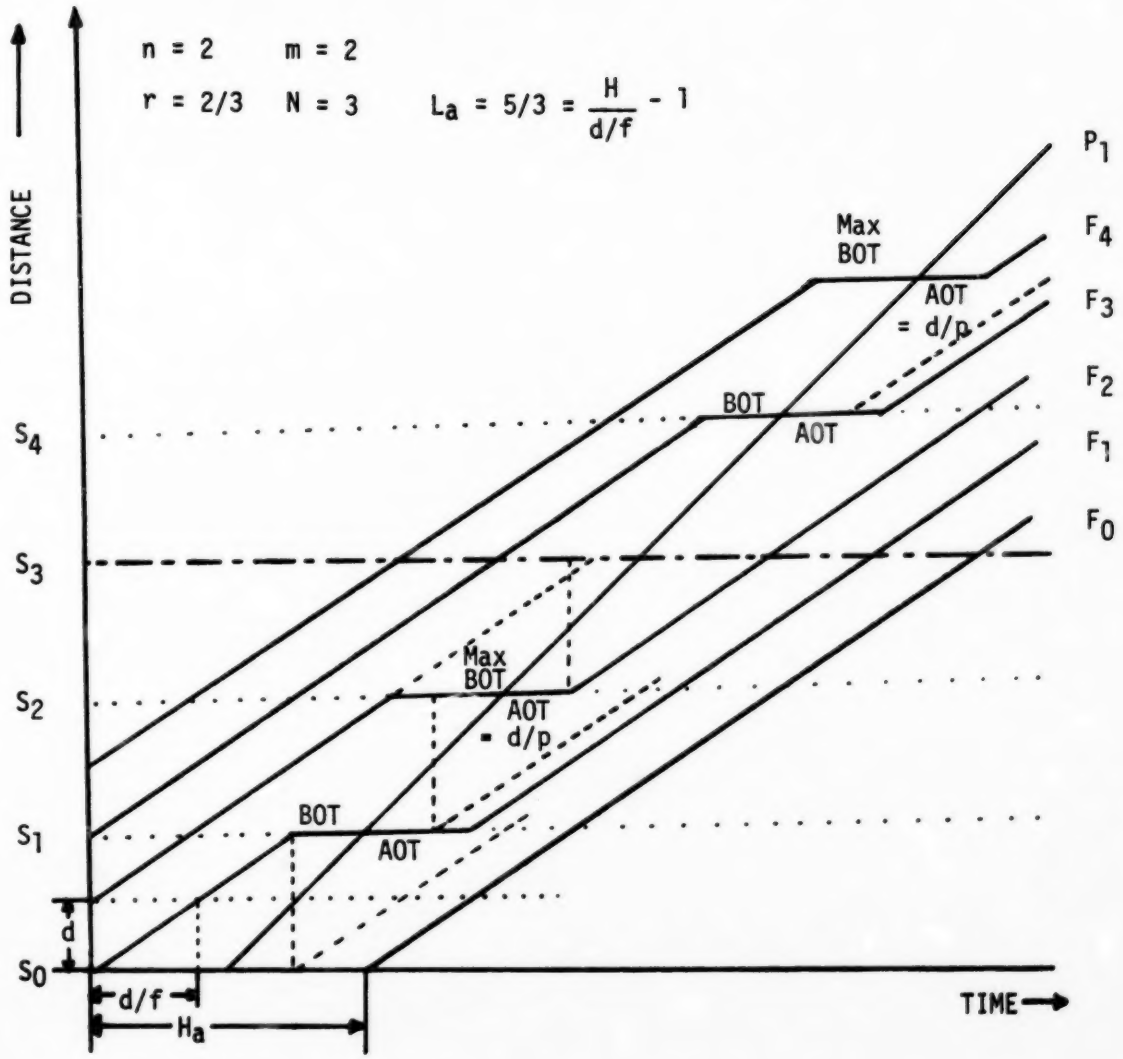


Figure 5 : Loss of Freight Trains when n = 2

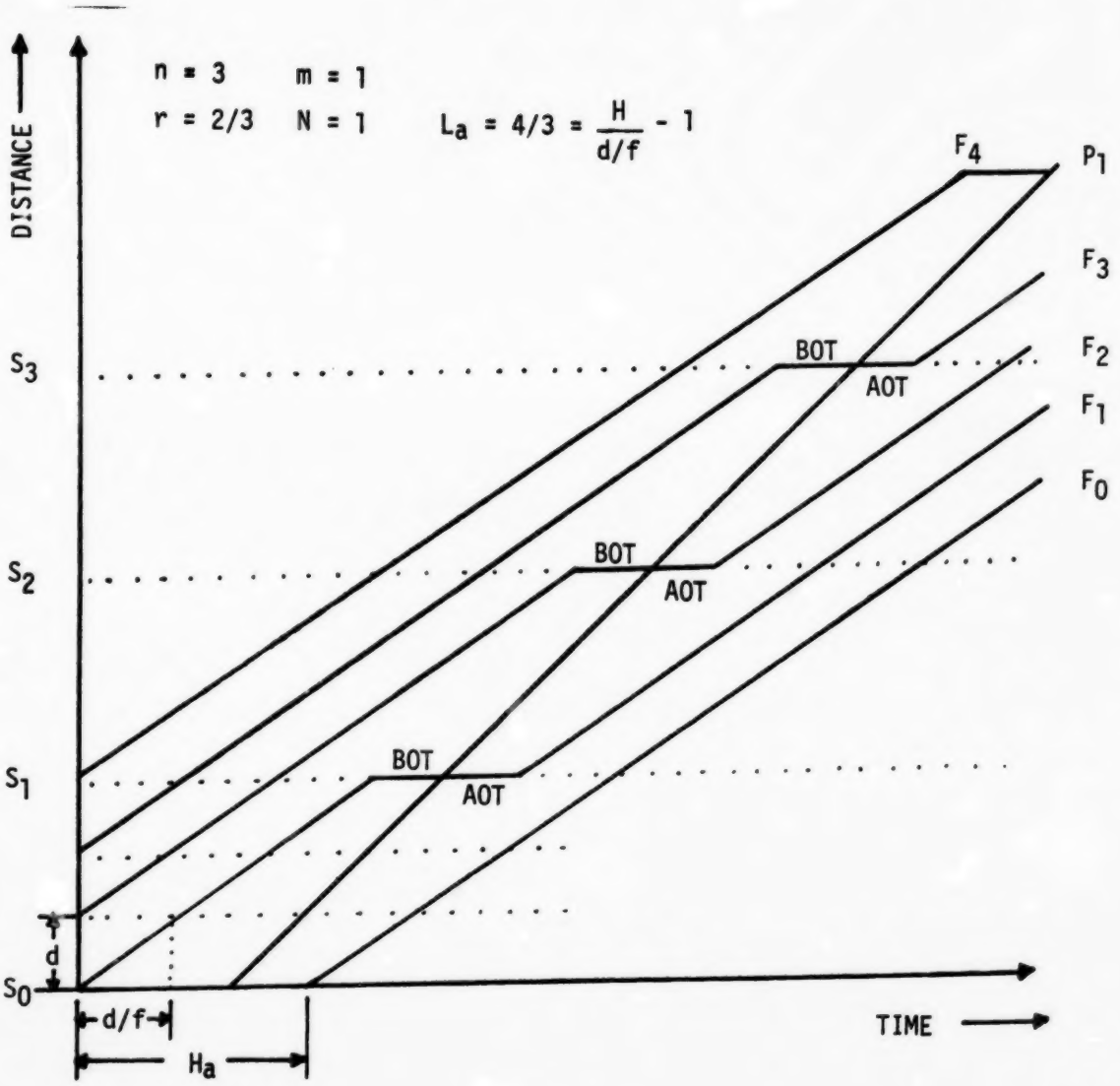


Figure 6 : Loss of Freight Trains when n = 3

Loss as a Function of r

Since 2 - n is positive when n = 1,

zero when n = 2 and

negative when n > = 3,

we have the actual loss tending to increase with the speed ratio when n = 1, tending to remain steady with the speed ratio when n = 2 and tending to decrease with speed ratio when n > = 3.

Intuition would normally rule that as r is smaller, the number of overtakes would be larger and consequently the greater the loss. This situation is exhibited only when n > = 3. When n = 1, we get a counter-intuitive result, which is what makes the analysis interesting. Given a saturated section of trains at a speed f, when n = 1, if we now wanted to introduce a faster train at speed p, then the higher the p, the better. To put it another way, if we wanted to increase the speed f, the equivalent loss goes up when n = 1. But then this is not a negative feature in the overall perspective since the final objective is to increase capacity. So, when f is increased, we get an increased capacity of running trains at speed f for the entire period in consideration, and the price in terms of equivalent trains lost due to the running of a train at speed p is a small one to pay.

For example,

if the f doubles to make r go from $\frac{5}{15}$ to $\frac{10}{15}$, the equivalent loss goes up from 1 to $1\frac{3}{5}$ - a mere price of $\frac{3}{5}$ th of a train when we can double the capacity. Table 1 gives values of L vs r when n = 1, n = 2 and n = 3.

Table 1 : Values of L versus r for n = 1, 2, 3

	n = 1	Loss	n = 2	Loss	n = 3	Loss
	$n(1-r)$	$2r + \frac{m-1}{N}$	$n(1-r)$	$2r + \frac{m-1}{N}$	$n(1-r)$	$2r + \frac{m-1}{N}$
$r = \frac{1}{8}$	$\frac{7}{8}$	$\frac{2}{8} + \frac{6}{8} = \frac{8}{8}$	$\frac{14}{8} = \frac{7}{4}$	$\frac{2}{8} + \frac{6}{4} = \frac{14}{8}$	$\frac{21}{8}$	$\frac{2}{8} + \frac{20}{8} = \frac{22}{8}$
$r = \frac{2}{8} = \frac{1}{4}$	$\frac{3}{4}$	$\frac{2}{4} + \frac{2}{4} = 1$	$\frac{6}{4} = \frac{3}{2}$	$\frac{2}{4} + \frac{2}{2} = \frac{6}{4}$	$\frac{9}{4}$	$\frac{2}{4} + \frac{8}{4} = \frac{10}{4}$
$r = \frac{3}{8}$	$\frac{5}{8}$	$\frac{6}{8} + \frac{4}{8} = \frac{10}{8}$	$\frac{10}{8} = \frac{5}{4}$	$\frac{6}{8} + \frac{4}{4} = \frac{14}{8}$	$\frac{15}{8}$	$\frac{6}{8} + \frac{14}{8} = \frac{20}{8}$
$r = \frac{4}{8} = \frac{1}{2}$	$\frac{1}{2}$	$1 + 0 = 1$	1	$1 + 0 = 1$	$\frac{3}{2}$	$1 + 1 = 2$
$r = \frac{5}{8}$	$\frac{3}{8}$	$\frac{10}{8} + \frac{2}{8} = \frac{12}{8}$	$\frac{3}{4}$	$\frac{10}{8} + \frac{2}{4} = \frac{14}{8}$	$\frac{9}{8}$	$\frac{10}{8} + 1 = \frac{18}{8}$
$r = \frac{6}{8} = \frac{3}{4}$	$\frac{1}{4}$	$\frac{6}{4} + \frac{2}{4} = 1$	$\frac{1}{2}$	$\frac{6}{4} + 0 = \frac{6}{4}$	$\frac{3}{4}$	$\frac{6}{4} + \frac{2}{4} = 2$
$r = \frac{7}{8}$	$\frac{1}{8}$	$\frac{14}{8} + 0 = \frac{14}{8}$	$\frac{1}{4}$	$\frac{14}{8} + 0 = \frac{14}{8}$	$\frac{3}{8}$	$\frac{14}{8} + \frac{2}{8} = \frac{16}{8} = 2$

7. Introducing More Than One Passenger Train

If more than one passenger train is to be introduced, then scheduling of these trains has an impact on the total number of freight trains lost. Say we are considering two passenger trains. If they are run apart such that at least one freight train can be scheduled between them (Figure 7), then the equivalent loss

$$L_1 = 2L_A$$

$$= 2\left(2r + \frac{m-1}{N}\right) \qquad \text{(from 16)}$$

$$= 4r + \frac{2(m-1)}{N} \qquad (20)$$

If they are run one after the other (Figure 8), then the headway between the affected pair of consecutive trains

$$H_2 = \frac{d}{p} + \frac{m-1}{N}\left(\frac{d}{f}\right) + \frac{d}{p} + \frac{d}{f} + \frac{d}{p}$$

$$= \frac{3d}{p} + \frac{m+N-1}{N}\left(\frac{d}{f}\right) \qquad (21)$$

The equivalent loss

$$L_2 = \frac{H_2}{\frac{d}{f}} = 1$$

$$= 3r + \frac{m-1}{N} \qquad (22)$$

Obviously $L_2 < L_1$. So, if more than one passenger train is to be introduced, best results are obtained if the passenger trains are run one after the other.

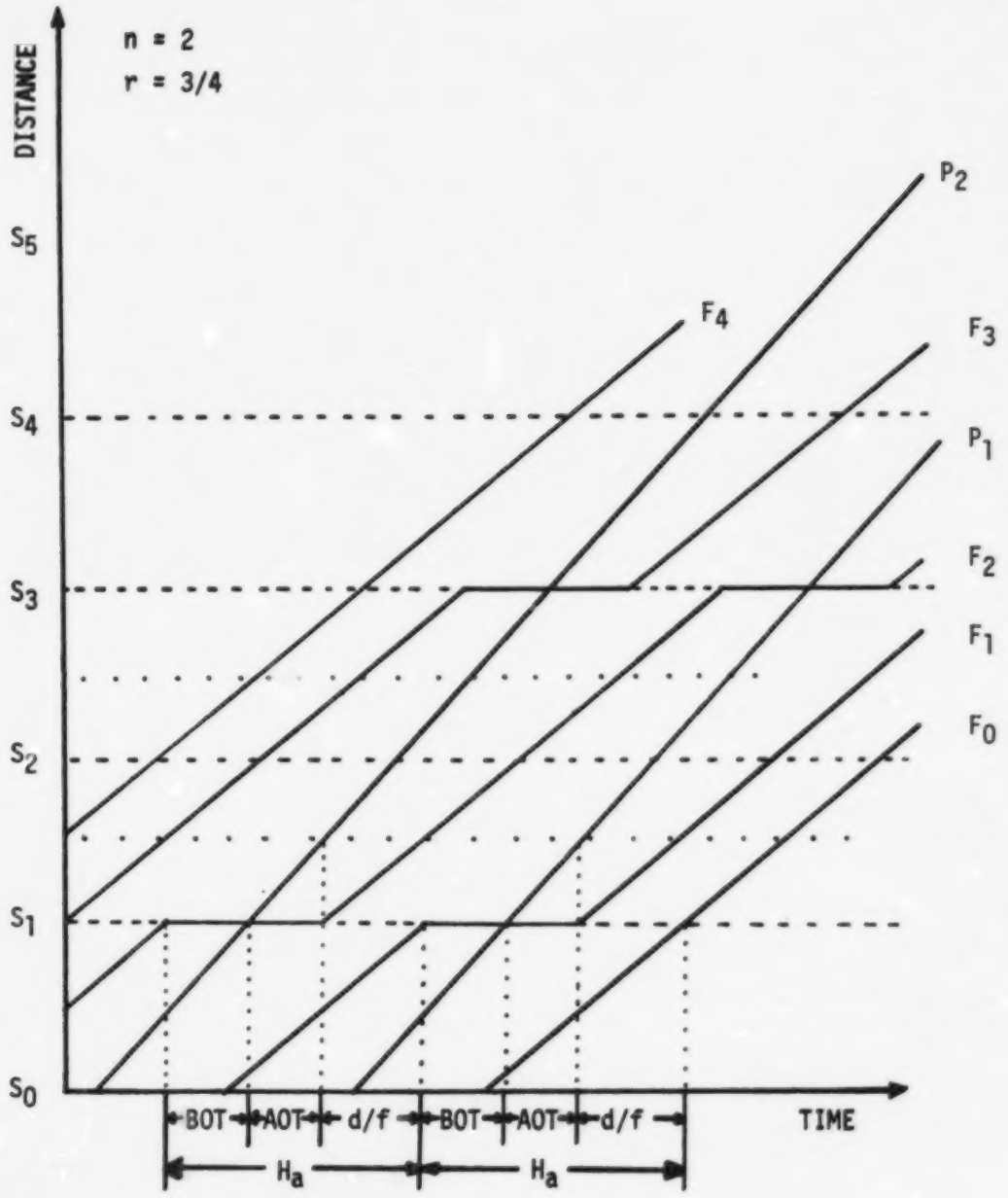


Figure 7 : Two Passenger Trains Scheduled Apart

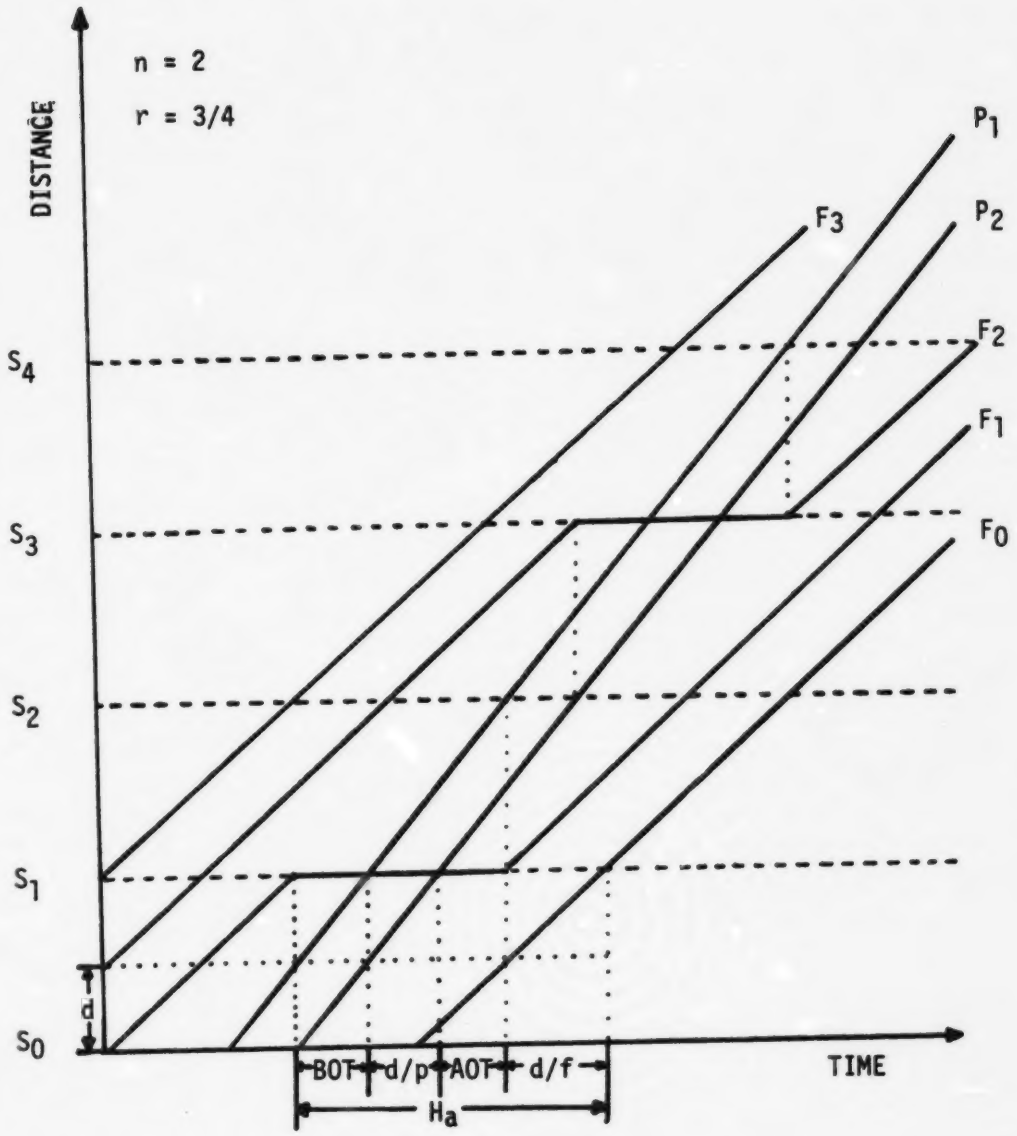


Figure 8 : Two Passenger Trains Scheduled One After the Other

Thus far, our assumption has been that scheduling of passenger trains is in our control. Suppose the schedules are exogenously specified, then it is interesting to know what the worst can be. While running one passenger train, the maximum headway is obtained when the BOT is just less than the BOT_{UB}. Consequently in the limit, we have the worst loss equal to L_{UB} (8). If we now consider two passenger trains with no control over their schedules (except that they have a linear path), then the worst loss is

$$\begin{aligned} 2L_{UB} &= 2n - 2(n-2)r \\ &= 4r + \frac{2m}{N} \end{aligned} \tag{23}$$

The above loss is incurred both when the passenger trains are run consecutively, but farthest apart such that a freight train cannot be scheduled in between and when the passenger trains are run apart.

We can now generalize for ℓ fast trains. Depending on the schedule selected, the least loss is given as

$$(\ell + 1)r + \frac{m-1}{N} \tag{24}$$

The worst loss is given as

$$2\,r + \frac{\ell m}{N} \tag{25}$$

Where m,N are given by (11).

In this section, we have examined one extension of our basic model. Other extensions are possible, as we relax the assumptions made in Section 2.

8. Conclusions and Directions for Future Research

The impact of Passenger Train Operation on Freight Train Operation is modeled in this paper. It is concluded that

(i) the speed ratio between the trains (r) and the number of blocks between consecutive siding (n) affect the freight train capacity. Calling the reduction in capacity as a loss, we conclude
- the loss generally increases as a function of n.
- the loss as a function of p is positive when $n = 1$, constant when $n = 2$ and negative when $n > 3$.

(ii) The case of more than one passenger train is an easy extension.

The questions for future research are :

(i) What if there are trains of more than one speed category?

(ii) How can the stochastic nature of train travel times be taken into account, along with braking and acceleration times, and stopping times?

(iii) What happens if the assumptions of equal block length and equal number of blocks between consecutive sidings are relaxed?

REFERENCES

FRANK, O. (1966), "Two-Way Traffic on a Single Line of Railway." Operations Research 14, 801-811.

MATERNINI, Matteo (1969), "Considerations Regarding Increasing the Commercial Train Speed." Monthly Bulletin of the International Railway Congress Association 46, 217-261.

PETERSEN, E.R. (1977), "Capacity of a Single-Track Rail Line." School of Business, Queen's University at Kingston, Ontario. Working Paper 77-38. Also CIGGT Report No. 78-1.

PETERSEN, E.R. and TAYLOR, A.J. (1983), "Line Block Prevention in Rail Line Dispatch and Simulation Models." INFOR 21.

POTTHOFF, G. (1968), "Timetable Networks with Very Fast Trains and Slow Trains." Monthly Bulletin of the International Railway Congress Association 45, 461-470.

RAGHURAM, G. (1979), "Effect on Operation Due to Speed Between Passenger Trains and Goods (Freight) Trains - Evaluation of a Model." Indian Institute of Management, Ahmedabad.

RAGHURAM, G. (1984), "Rail Freight Operations and the Classification Policy - Models and Solution Procedures." Unpublished Ph.D. Thesis, Northwestern University.

SINGH, J. (1965), Statistical Aids to Railway Operation, Asia Publishing House, Bombay, 97-100.

STAHLI, S. (1969), "Basic Problem of Timetable Compilation." Monthly Bulletin of the International Railway Congress Association 46, 641-652.

VOROBJEV, N.A. (1968), "Influence of High Speed Passenger Traffic on the Graphic Timetable and Capacity of the Lines." Monthly Bulletin of the International Railway Congress Association 45, 555-572.